Water is added to the surface of vertical soil column (filled with a homogeneous soil) at a steady rate of 1 cm/s (= 1 $cm^3/cm^2/s$). The flow is steady (does not vary with time), and the soil is unsaturated.

If the column is 10 cm long, what is K? Assume the column is free-draining at the lower end.

Answer: next page:

$q = -K\, dH/dz$.

As mentioned in the question, the soil is unsaturated, i.e., the steady flow at the surface is insufficient to saturate the soil (similar to rainfall that does not produce water on the surface of the soil). That is, the flow is unsaturated (flow takes place in smaller pores, larger pores are empty).

Take z = 0 as the top of the column, and assume the z axis points downwards. Then:

$$H = h - z$$

At z = 0, what is h? We don't know exactly, call this value h_1 cm

$$H(0) = 0 + h_1 = h_1 \text{ cm}$$

At z = 10 cm, the water is leaving the column and again $h = h_1$, since the flow is steady (means that everywhere in the column the same smaller pore sizes are filled with water, and the same larger pore sizes are empty), so

$$H(10 \text{ cm}) = -10 + h_1 \text{ cm}$$

Thus, $h = h_1$ everywhere and K is the value of K at that value of h, i.e.,

$$q = -K(h_1)\, dH/dz = -K(h_1)\, (H(10) - H(0))/(10 - 0) = -K(h_1)\, (-10 + h_1 - h_1)/10$$

i.e.,

$$q = K(h_1)$$

Note 1: in the above calculation, H is linear in z, and so we need H at only 2 points to compute dH/dz. The reason for this is that, in this case, $K(h_1)$ is constant and uniform, since h_1 is constant.

Note 2: $h = h_1$ is constant everywhere in this example, meaning that flow is driven by the gravitational head alone, not pressure head.

Note 3: If we measure (e.g., using a tensiometer) h_1, then we have just measured one point on the K(h) curve.

Note 4: Following from 3, this calculation shows that, for steady unsaturated flow in a vertical column (with a homogeneous soil), we can measure K(h) by taking different values of q, and waiting for steady state flow conditions, and measuring the value of h associated with the selected q.